MATH 1316 Section 7.1 Lial, 11^{th} ed.

1. Discuss attached chart of "Facts About Triangles"
2. Oblique $\Delta \Rightarrow$ not a right Δ.
3. Data required for solving an oblique Δ:
any 3 parts, at least one of which is a side.
Specifically:
(a) 1 side & 2 angles (ASA or SAA) (used to be ASA only)
(b) 2 sides & 1 included angle (SAS)
(c) 2 sides & 1 angle not included (ambiguous case)
(d) 3 sides

4. Conjecture: In ΔABC, $\frac{a}{A} = \frac{b}{B} = \frac{c}{C}$.
This is wrong, but it's close to the Law of Sines.

5. Law of Sines: In ΔABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

6. Discuss attached chart for solving ASA or SAA.
Note: All other cases require the Law of Cosines.

Facts About Triangles

1. For each angle θ in any triangle, $0° < m(\angle\theta) < 180°$.

2. The sum of the angles of any triangle equals 180°.

3. In any triangle ΔABC,
   with angles $\angle A$, $\angle B$, and $\angle C$ and
   corresponding sides a, b, and c,
   $a < b < c$ if and only if $m(\angle A) < m(\angle B) < m(\angle C)$.

   In other words, in any triangle
   the shortest side is opposite the smallest angle,
   the medium length side is opposite the medium size angle,
   the longest side is opposite the largest angle.

4. A triangle has **at most** one angle of measure greater than or equal to 90°.

5. If a triangle has an angle of measure greater than or equal to 90°,
   it must be the largest angle (see #4 above), and
   therefore must be opposite the longest side (see #3 above).

Solving Triangles Using the
<u>Law of Sines and Law of Cosines</u>

1. Solving Triangles by the <u>ASA or SAA Methods</u>

Note: One side and two angles are given.

1. Find the third angle by subtracting the two given angles from 180°.
2. Use the given side, the corresponding angle, and either of the two remaining angles to find a second side with the Law of Sines.
3. Use either of the two known sides, the corresponding angle, and the angle opposite the unknown side to find the third side with the Law of Sines.

2. Solving Triangles by the <u>SAS Method</u>

Note: Two sides and the included angle are given

1. Use the Law of Cosines to find the third side.
2. Use the Law of Sines to find the remaining angle opposite the <u>smaller corresponding side</u>.

   Note: Since this side is <u>not</u> the longest side, then the corresponding angle must be acute, thus avoiding the ambiguous case of the Law of Sines.

3. Find the last angle by subtracting the other two from 180°.

3. Solving Triangles by the <u>SSS Method</u>

Note: Three sides are given

1. Use the Law of Cosines to find the <u>angle opposite the longest side</u>.

   Note: If the triangle has an angle of 90° or more, it has only one such angle, and it must be opposite the longest side. Since $\cos^{-1}$ has a range of 0° to 180°, the Law of Cosines will identify such an angle.

2. Use the Law of Sines to find either one of the remaining angles.

   Note: Since the largest angle was found in step 1, the other two angles must be acute, thus avoiding the ambiguous case of the Law of Sines.

3. Find the last angle by subtracting the other two from 180°.

7. Given $\angle A = 40^\circ$, $\angle B = 105^\circ$, $b = 20.3$ cm.
Solve $\triangle ABC$.

1. $\angle C = 180^\circ - 40^\circ - 105^\circ = 35^\circ$

2. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} \Rightarrow \dfrac{a}{\sin 40^\circ} = \dfrac{20.3\text{ cm}}{\sin 105^\circ} \Rightarrow$

$$a = \frac{20.3\text{ cm} \cdot \sin 40^\circ}{\sin 105^\circ} \Rightarrow a = 13.509\text{ cm.}$$

3. $\dfrac{c}{\sin C} = \dfrac{b}{\sin B} \left(\text{OR} = \dfrac{a}{\sin A}\right) \Rightarrow$

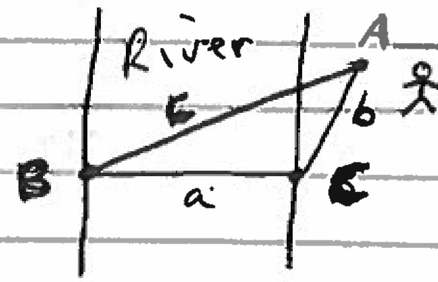
$$\frac{c}{\sin 35^\circ} = \frac{20.3\text{ cm}}{\sin 105^\circ} \Rightarrow c = \frac{20.3\text{ cm} \cdot \sin 35^\circ}{\sin 105^\circ} \Rightarrow$$

$c = 12.054$ cm.

8. Measuring the Width of a River

Suppose you are by a river with equipment to measure distances and angles, but you can't measure distance across the river. Pick a point A on your side of the river. Then pick two points on the bank of the river directly across the river from each other, say points B & C.

Points A, B, C form a triangle, and the length of side "a" is the width of the river.



Since side b is on your side, then measure $b = 347.6$ ft.

Also $\angle A$ and $\angle C$ are on your side, so measure $\angle A = 31.1°$ and $\angle C = 112.9°$.

Then $\angle B = 180° - 31.1° - 112.9° = 36°$.

Using the Law of Sines, we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{a}{\sin 31.1°} = \frac{347.6\text{ft}}{\sin 36°} \Rightarrow$$

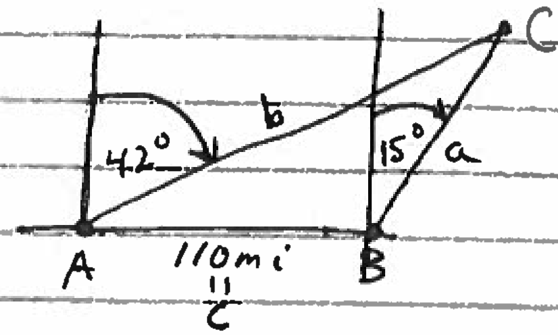
$$a = \frac{347.6\text{ ft} \cdot \sin 31.1°}{\sin 36°} = 305.5\text{ ft}.$$

Hence the river has width $a = 305.5$ ft.

9. Station A is 110 miles west of Station B. Station A spots a fire on a bearing of N42°E. Station B spots the fire on a bearing of N15°E. How far is the fire from Station A.

Let's call the location of the fire point C.

$\angle A = 90° - 42° = 48°$,
$\angle B = 90° + 15° = 105°$,
$\angle C = 180° - 48° - 105° = 27°$



Using the Law of Sines, we have

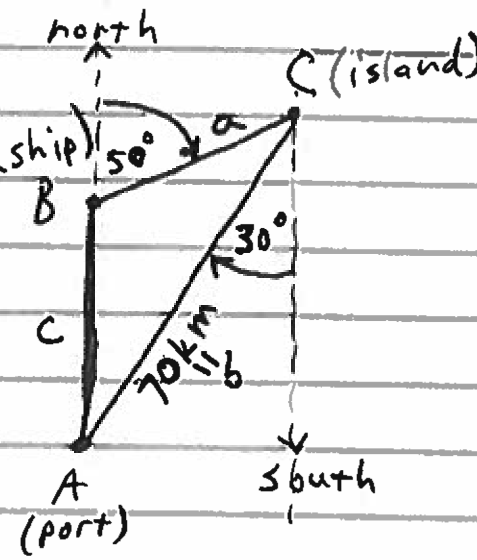
$$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{b}{\sin 105°} = \frac{110\,mi}{\sin 27°} \Rightarrow$$

$$b = \frac{110\,mi \cdot \sin 105°}{\sin 27°} = 234.04\ mi.$$

Hence the fire is 234.04 miles from Station A.

10. A port is located 70 km S30°W of an island. A ship sails due north from the port. At some point the navigator of the ship spots the island on a bearing of N50°E. How far is the ship from the port at this time?

Let the ship be point B, the port be point A, and the island point C.



∠A is an alternate interior angle with the 30° angle, so ∠A = 30°.
$\angle B = 180° - 50° = 130°$, and so
$\angle C = 180° - 30° - 130° = 20°$.

Using the Law of Sines, we have

$$\frac{c}{\sin C} = \frac{b}{\sin B} \Rightarrow \frac{c}{\sin 20°} = \frac{70\text{ km}}{\sin 130°} \Rightarrow$$

$$c = \frac{70\text{ km} \cdot \sin 20°}{\sin 130°} = 31.25\text{ km}.$$

Hence the distance from the ship to the port is $c = 31.25$ km.

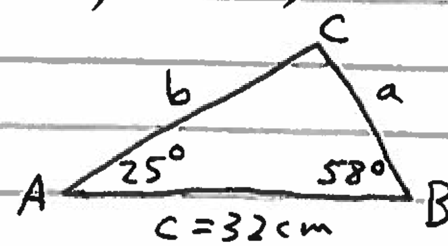
11. We now present an application for computing the area of a triangle when given any 2 sides and the angle between them.

The area of ΔABC with sides a, b, c is:

$$\text{Area} = \frac{1}{2}ab \cdot \sin C = \frac{1}{2}ac \cdot \sin B = \frac{1}{2}bc \cdot \sin A.$$

In other words, the area of ΔABC is $\frac{1}{2}$ the product of any 2 sides times the sine of the angle between those 2 specific sides.

12. Determine the area of ΔABC if $\angle A = 25°$, $\angle B = 58°$, $c = 32$ cm.



$\angle C = 180° - 25° - 58° = 97°$

Using the Law of Sines, we have

$$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{b}{\sin 58°} = \frac{32 \text{ cm}}{\sin 97°} \Rightarrow$$

$$b = \frac{32 \text{ cm} \cdot \sin 58°}{\sin 97°} = 27.34 \text{ cm}.$$

Hence $\text{Area} = \frac{1}{2}bc \cdot \sin A = \frac{1}{2}(27.34 \text{ cm})(32 \text{ cm}) \sin 25° \Rightarrow$

$$\text{Area} = 184.87 \text{ cm}^2.$$