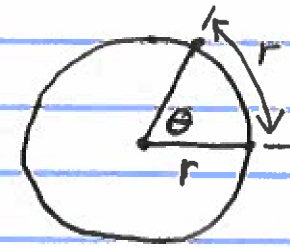
Section 3.1

1. We now introduce radian measure of an angle. In a circle, one radian is the measure of a central angle that intercepts an arc whose length is the radius of the circle.



In the diagram, the central angle θ intercepts (subtends) an arc whose length is the radius r. Therefore, $m(\angle\theta) = 1$ radian, denoted 1 rad.

2. Conversion between degrees and radians: The circumference of a circle is $C = 2\pi r$. Since a central angle of 1 rad subtends an arc of length 1 radius r, then there are 2π radians in a full circle. Since a full circle also has $360°$, then 2π rad $= 360°$. Reducing this yields π rad $= 180°$. Thus $1 \text{ rad} = \frac{180°}{\pi} \approx 57.3°$. The relationship π rad $= 180°$ is used to convert between degrees and radians.

3. Examples:

(a) $4.2 \text{ rad} = 4.2 \text{ rad} \cdot \frac{180°}{\pi \text{ rad}} = 240.64°$ (rounded).

(b) $338.7° = 338.7° \cdot \frac{\pi \text{ rad}}{180°} = 5.91 \text{ rad}$ (rounded).

Note: When an angle measure is given with no units specified, the default is radians.

Table of Exact Trigonometric Values

θ (rad/deg)	$0 = 0°$	$\frac{\pi}{6} = 30°$	$\frac{\pi}{4} = 45°$	$\frac{\pi}{3} = 60°$	$\frac{\pi}{2} = 90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0
$\sec\theta$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	∞
$\csc\theta$	∞	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1

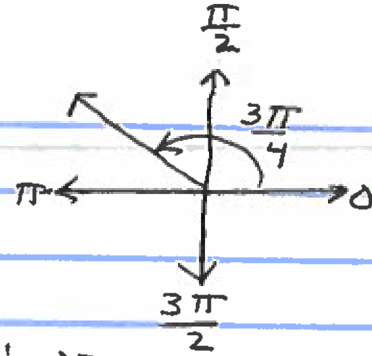
4. Referring to the Table of Exact Values, this conversion factor between degrees and radians yields $0° = 0$ rad, $30° = \frac{\pi}{6}$ rad, $45° = \frac{\pi}{4}$ rad, $60° = \frac{\pi}{3}$ rad, and $90° = \frac{\pi}{2}$ rad. Note that these equalities are exact, not rounded.

5. Even when seeking an approximate answer via a calculator, it is important to use the π key on the calculator to maximize the precision. That is, do not use 3.14 or $\frac{22}{7}$ for π when converting between degrees and radians.

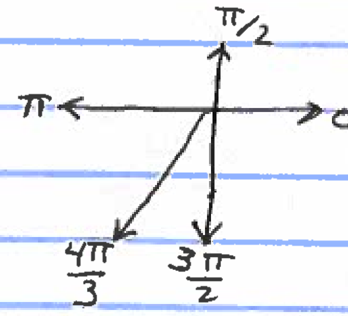
Also, it is useful to extend the radian measure in the Table of Exact Values and memorize radian measure for primary angles up to 360°. For example, $180° = \pi$ rad, $270° = \frac{3\pi}{2}$ rad, $225° = \frac{5\pi}{4}$ rad, $330° = \frac{11\pi}{6}$ rad, and so on.

6. Computing exact trig values with radians functions basically the same as with degrees. However, the calculation of the reference angle involves fractions of π instead of integers as in degrees.

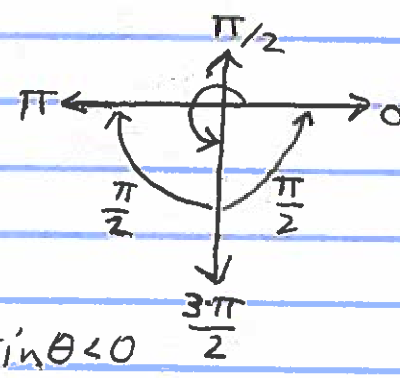
Examples:



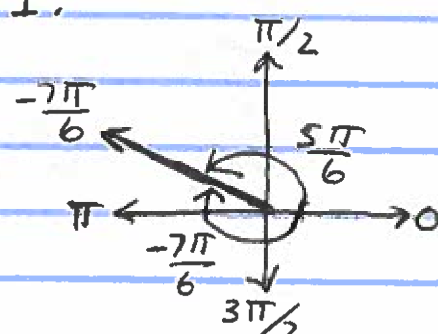
(a) For $\cos\frac{3\pi}{4}$, the reference angle is $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$. Also, $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$. Finally, $\frac{3\pi}{4}$ is in QII and $\cos\theta < 0$ in QII. Therefore $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$.



(b) For $\tan\frac{4\pi}{3}$, the reference angle is $\frac{4\pi}{3} - \pi = \frac{\pi}{3}$, and $\tan\frac{\pi}{3} = \sqrt{3}$. Also, $\frac{4\pi}{3}$ is in QIII where $\tan\theta > 0$. Thus $\tan\frac{4\pi}{3} = \sqrt{3}$.

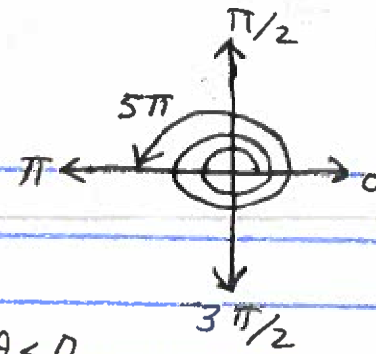


(c) Since $\frac{3\pi}{2}$ terminates on the y-axis, then the reference angle is $\frac{\pi}{2}$, and $\sin\frac{\pi}{2} = 1$. Also, $\frac{3\pi}{2}$ is between QIII & QIV, and $\sin\theta < 0$ in QIII & QIV. Thus $\sin\frac{3\pi}{2} = -1$.

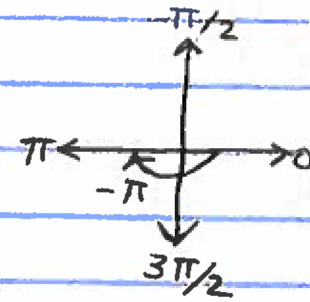


(d) $-\frac{7\pi}{6}$ is coterminal with $\frac{5\pi}{6}$, so the reference angle is $\pi - \frac{5\pi}{6} = \frac{\pi}{6}$, and $\csc\frac{\pi}{6} = 2$. Also, $-\frac{7\pi}{6}$ terminates in QII, where $\csc\theta > 0$. Thus $\csc\left(-\frac{7\pi}{6}\right) = 2$.

(e) Since 5π terminates on the negative x-axis, the reference angle is 0, and $\sec 0 = 1$. Since 5π is between QII & QIII and $\sec\theta < 0$ in QII & QIII, then $\sec 5\pi = -1$.



(f) Since $-\pi$ terminates on the x-axis, the reference angle is 0. Since $\cot 0 = \infty$, then $\cot(-\pi) = \infty$. That is, $\cot(-\pi)$ is undefined.



7. When computing approximate trig values on a calculator, the only difference from degrees is to make sure the calculator is in radian mode.

Examples: With calculator in radian mode, we have:

(a) $\sin(2) = .9092974268$.

(b) $\cos\left(\frac{2\pi}{5}\right) = .3090169944$ (using the π key).

(c) $\cot(-10.3) = \frac{1}{\tan(-10.3)} = -.8347508961$.

(d) $\sec\left(\frac{\pi}{2}\right) = \frac{1}{\cos(\pi/2)}$ produces the message "ERR: DIVIDE BY 0" or something equivalent. This is interpreted as $\sec\left(\frac{\pi}{2}\right)$ is undefined, so $\sec\left(\frac{\pi}{2}\right) = \infty$.