Section 8.3

1. Multiplying complex numbers in trigonometric form.

$[r_1 \cdot (\cos\theta_1 + i \cdot \sin\theta_1)] \cdot [r_2 \cdot (\cos\theta_2 + i \cdot \sin\theta_2)] =$

$$(r_1 r_2) \cdot \left(\cos(\theta_1+\theta_2) + i \cdot \sin(\theta_1+\theta_2)\right).$$

That is, simply <u>multiply</u> the magnitudes r_1 & r_2, and <u>add</u> the angles θ_1 & θ_2.

Examples:

(a) $[2 \cdot (\cos 100° + i \cdot \sin 100°)] \cdot [5 \cdot (\cos 130° + i \cdot \sin 130°)] =$

$(2 \cdot 5) \cdot (\cos(100° + 130°) + i \cdot \sin(100° + 130°)) =$

$10 \cdot (\cos 230° + i \cdot \sin 230°)$.

(b) $[3 \cdot (\cos\frac{\pi}{4} + i \cdot \sin\frac{\pi}{4})] \cdot [6.1(\cos\frac{3\pi}{4} + i \cdot \sin\frac{3\pi}{4})] =$

$(3 \cdot 6.1)\left(\cos\left(\frac{\pi}{4}+\frac{3\pi}{4}\right) + i \cdot \sin\left(\frac{\pi}{4}+\frac{3\pi}{4}\right)\right) =$

$18.3(\cos\pi + i \cdot \sin\pi)$.

<u>Note</u>: If $\theta_1+\theta_2 \geq 360°$ (or $\geq 2\pi$) then reduce $\theta_1+\theta_2$ to an angle between $0°$ & $360°$ (or between 0 & 2π). by subtracting $360°$ (or subtracting 2π).

(c) $[2(\cos 200° + i \cdot \sin 200°)] \cdot [5(\cos 336° + i \cdot \sin 336°)] =$

$(2 \cdot 5) \cdot (\cos(200° + 336°) + i \cdot \sin(200° + 336°)) =$

$10 \cdot (\cos 536° + i \cdot \sin 536°) =$ (subtracting $536° - 360°$)

$10 \cdot (\cos 176° + i \cdot \sin 176°)$, where $0° \leq 176° < 360°$.

(d) $\left[3\left(\cos\frac{3\pi}{2}+i\cdot\sin\frac{3\pi}{2}\right)\right]\cdot\left[7\left(\cos\frac{4\pi}{5}+i\cdot\sin\frac{4\pi}{5}\right)\right]=$

$$(3\cdot 7)\cdot\left(\cos\left(\frac{3\pi}{2}+\frac{4\pi}{5}\right)+i\cdot\sin\left(\frac{3\pi}{2}+\frac{4\pi}{5}\right)\right)=$$

$$21\cdot\left(\cos\frac{23\pi}{10}+i\cdot\sin\frac{23\pi}{10}\right).$$

In order to determine if $\frac{23\pi}{10}<2\pi$, convert 2π to a fraction with denominator 10 (like $\frac{23\pi}{10}$).

So $2\pi=\frac{2\pi}{1}\cdot\frac{10}{10}=\frac{20\pi}{10}$.
Clearly $\frac{23\pi}{10}>\frac{20\pi}{10}$, so $\frac{23\pi}{10}>2\pi$.

Thus we must reduce $\frac{23\pi}{10}$ to a coterminal angle θ which satisfies $0\le\theta<2\pi$ by subtracting 2π from $\frac{23\pi}{10}$. Fortunately, we already converted 2π to the fraction $\frac{20\pi}{10}$ above.

Therefore $\theta=\frac{23\pi}{10}-2\pi=\frac{23\pi}{10}-\frac{20\pi}{10}=\frac{3\pi}{10}$.

Hence $\left[3\left(\cos\frac{3\pi}{2}+i\cdot\sin\frac{3\pi}{2}\right)\right]\cdot\left[7\left(\cos\frac{4\pi}{5}+i\cdot\sin\frac{4\pi}{5}\right)\right]=$

$$21\left(\cos\frac{23\pi}{10}+i\cdot\sin\frac{23\pi}{10}\right)\ \text{(from above)}=$$

$$21\left(\cos\frac{3\pi}{10}+i\cdot\sin\frac{3\pi}{10}\right),\ \text{where } 0\le\frac{3\pi}{10}<2\pi.$$

2. Dividing complex numbers in trigonometric form.

$$[r_1(\cos\theta_1 + i\cdot\sin\theta_1)] \div [r_2(\cos\theta_2 + i\cdot\sin\theta_2)] =$$

$$\frac{r_1(\cos\theta_1 + i\cdot\sin\theta_1)}{r_2(\cos\theta_2 + i\cdot\sin\theta_2)} = \left(\frac{r_1}{r_2}\right)\cdot\left(\cos(\theta_1-\theta_2) + i\cdot\sin(\theta_1-\theta_2)\right).$$

That is, simply <u>divide</u> the magnitudes r_1 & r_2, and <u>subtract</u> the angles <u>in the specific order indicated</u>.

Examples:

(a) $[24(\cos 350° + i\cdot\sin 350°)] \div [3(\cos 147° + i\cdot\sin 147°)] =$

$$\left(\frac{24}{3}\right)\cdot\left(\cos(350°-147°) + i\cdot\sin(350°-147°)\right) =$$

$$8\cdot(\cos 203° + i\cdot\sin 203°)$$

(b) $\dfrac{26.23\left(\cos\frac{5\pi}{4} + i\cdot\sin\frac{5\pi}{4}\right)}{6.1\left(\cos\frac{\pi}{6} + i\cdot\sin\frac{\pi}{6}\right)} =$

$$\left(\frac{26.23}{6.1}\right)\cdot\left(\cos\left(\frac{5\pi}{4}-\frac{\pi}{6}\right) + i\cdot\sin\left(\frac{5\pi}{4}-\frac{\pi}{6}\right)\right) =$$

$$4.3\left(\cos\frac{13\pi}{12} + i\cdot\sin\frac{13\pi}{12}\right).$$

<u>Note</u>: The least common denominator of $\frac{5\pi}{4}$ & $\frac{\pi}{6}$ is 12, so

$$\frac{5\pi}{4} - \frac{\pi}{6} = \frac{5\pi}{4}\cdot\frac{3}{3} - \frac{\pi}{6}\cdot\frac{2}{2} = \frac{15\pi}{12} - \frac{2\pi}{12} = \frac{13\pi}{12}.$$

Note: If $\theta_1 - \theta_2 < 0°$ (or < 0 radians) then convert $\theta_1 - \theta_2$ to a coterminal angle between $0°$ and $360°$ (or between 0 and 2π) by adding $360°$ (or adding 2π).

(c) $$\frac{10(\cos 45° + i \cdot \sin 45°)}{2(\cos 60° + i \cdot \sin 60°)} = \left(\frac{10}{2}\right) \cdot \left(\cos(45° - 60°) + i \cdot \sin(45° - 60°)\right) =$$

$$5 \cdot \left(\cos(-15°) + i \cdot \sin(-15°)\right) = (\text{adding } -15° + 360°)$$
$$5 \cdot (\cos 345° + i \cdot \sin 345°).$$

(d) $$\frac{8.06\left(\cos \frac{\pi}{2} + i \cdot \sin \frac{\pi}{2}\right)}{2.6\left(\cos \frac{4\pi}{3} + i \cdot \sin \frac{4\pi}{3}\right)} =$$

$$\left(\frac{8.06}{2.6}\right) \cdot \left(\cos\left(\frac{\pi}{2} - \frac{4\pi}{3}\right) + i \cdot \sin\left(\frac{\pi}{2} - \frac{4\pi}{3}\right)\right) =$$

(getting a common denominator)

$$3.1 \cdot \left(\cos\left(\frac{\pi}{2} \cdot \frac{3}{3} - \frac{4\pi}{3} \cdot \frac{2}{2}\right) + i \cdot \sin\left(\frac{\pi}{2} \cdot \frac{3}{3} - \frac{4\pi}{3} \cdot \frac{2}{2}\right)\right) =$$

$$3.1\left(\cos\left(\frac{3\pi}{6} - \frac{8\pi}{6}\right) + i \cdot \sin\left(\frac{3\pi}{6} - \frac{8\pi}{6}\right)\right) =$$

$$3.1\left(\cos \frac{-5\pi}{6} + i \cdot \sin \frac{-5\pi}{6}\right) =$$

(converting to a coterminal angle)

$$3.1\left(\cos\left(\frac{-5\pi}{6} + 2\pi\right) + i \cdot \sin\left(\frac{-5\pi}{6} + 2\pi\right)\right) =$$

$$3.1\left(\cos\left(\frac{-5\pi}{6} + \frac{12\pi}{6}\right) + i \cdot \sin\left(\frac{-5\pi}{6} + \frac{12\pi}{6}\right)\right) =$$

$$3.1\left(\cos \frac{7\pi}{6} + i \cdot \sin \frac{7\pi}{6}\right), \text{ where } 0 \le \frac{7\pi}{6} < 2\pi.$$