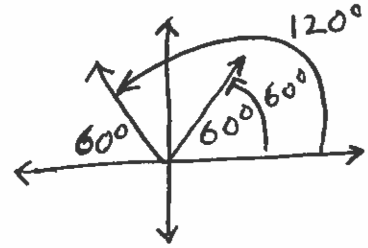
Section 6.2

1. Solve for x $(0° \le x < 360°)$.

$3\sin x = \sqrt{3} + \sin x \Rightarrow 2\sin x = \sqrt{3} \Rightarrow \sin x = \frac{\sqrt{3}}{2} = +\frac{\sqrt{3}}{2}$.

$x_{ref} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$; $\sin x > 0$ in QI & QII

$\boxed{x = 60°, 120°}$



2. Solve for x $(0° \le x < 360°)$.

$\sin x \cdot \tan x - \sin x = 0 \Rightarrow$

$\sin x \cdot \tan x = \sin x \Rightarrow$

$\sin x \cdot (\tan x - 1) = 0 \Rightarrow \sin x = 0$ or $\tan x - 1 = 0 \Rightarrow$

$\sin x = 0$ or $\tan x = 1$.

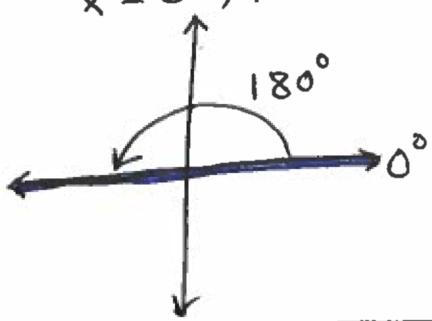
$\sin x = 0 \Rightarrow$

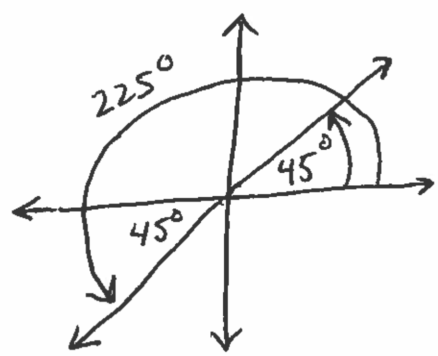
$x_{ref} = \sin^{-1}(0) = 0°$

$x = 0°, 180°$

$\tan x = 1 \Rightarrow x_{ref} = \tan^{-1}(1) = 45°$

$\tan x > 0$ in QI, QIII $\Rightarrow x = 45°, 225°$





$\boxed{\text{Therefore } x = 0°, 180°, 45°, 225°}$

Note: Check these solutions in the original equation. All four solutions are valid.

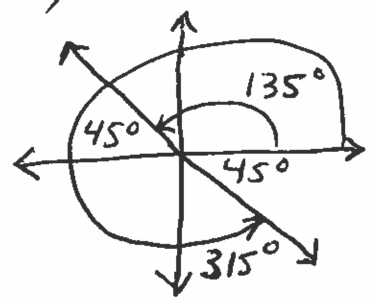
3. Solve for x $(0° \le x < 360°)$.

$\sin x + \cos x = 0 \Rightarrow \frac{\sin x}{\cos x} + \frac{\cos x}{\cos x} = \frac{0}{\cos x} \Rightarrow$

$\tan x + 1 = 0 \Rightarrow \tan x = -1.$

$x_{ref} = \tan^{-1}(1) = 45°$; $\tan x < 0$ in QII, QIV.

Thus $x = 135°, 315°$.



Note: Check solutions in the original equation since dividing by $\cos x$. Both solutions are valid.

4. Solve for x $(0 \le x < 2\pi)$ (radians).

$\tan x + \sqrt{3} = \sec x \Rightarrow (\tan x + \sqrt{3})^2 = (\sec x)^2 \Rightarrow$

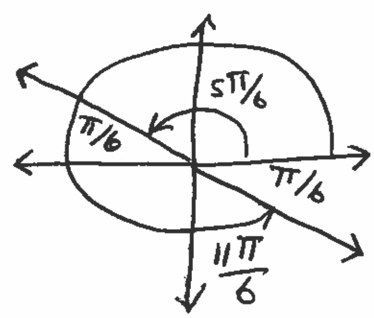
$\tan^2 x + 2\sqrt{3} \cdot \tan x + 3 = \sec^2 x \Rightarrow$

$\tan^2 x + 2\sqrt{3} \cdot \tan x + 3 = 1 + \tan^2 x$ (pythagorean identity) $\Rightarrow$

$2\sqrt{3} \cdot \tan x + 3 = 1 \Rightarrow 2\sqrt{3} \cdot \tan x = -2 \Rightarrow \tan x = \frac{-2}{2\sqrt{3}} = -\frac{\sqrt{3}}{3}.$

$x_{ref} = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$ (from table); $\tan x < 0$ in QII, QIV.

Therefore $x = \frac{5\pi}{6}, \frac{11\pi}{6}$.



Note: Check these in the original equation since squaring both sides of the equation.

$\frac{11\pi}{6}$ satisfies the equation but $\frac{5\pi}{6}$ does <u>NOT</u>.

Thus $\underline{x = \frac{11\pi}{6}}$ is the only solution for $0 \le x < 2\pi$.

5. Solve for x $(0 \le x < 2\pi)$ (radians).

$\tan^2 x + \tan x = 2 \Rightarrow \tan^2 x + \tan x - 2 = 0 \Rightarrow$

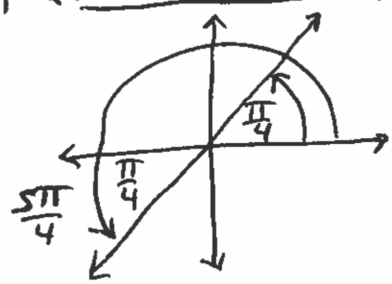
$(\tan x - 1)(\tan x + 2) = 0 \Rightarrow \tan x - 1 = 0$ or $\tan x + 2 = 0 \Rightarrow$

$\tan x = 1$ or $\tan x = -2$.

Case 1: $\tan x = 1 \Rightarrow x_{ref} = \tan^{-1}(1) = \frac{\pi}{4}$ (from the table).

Also, $\tan x > 0$ in QI, QIII.

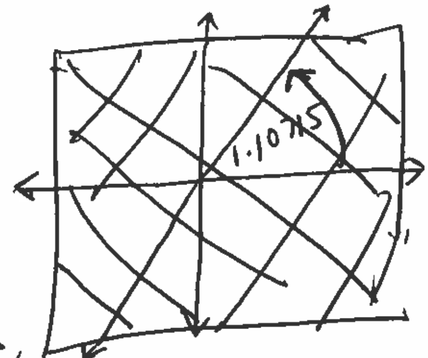
Thus $x = \frac{\pi}{4}, \frac{5\pi}{4}$.

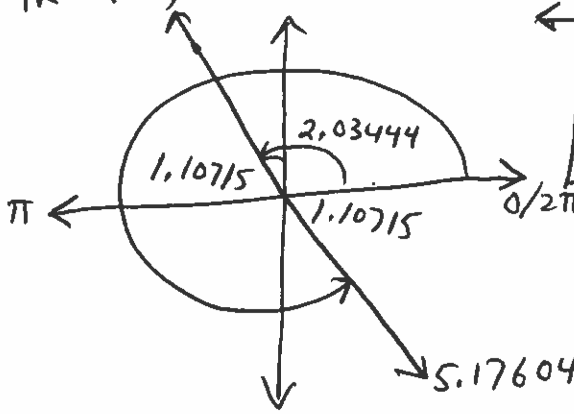


Case 2: $\tan x = -2 \Rightarrow$

$x_{ref} = \tan^{-1}(2) \approx 1.10715$ (radians).

Also, $\tan x < 0$ in QII, QIV.





Therefore $x = \pi - 1.10715 \approx 2.03444$ and

$x = 2\pi - 1.10715 \approx 5.17604$

Solutions: $x = \underbrace{\frac{\pi}{4}, \frac{5\pi}{4}}_{\text{exact solutions}}, \underbrace{2.03444, 5.17604}_{\text{approximate solutions}}$

6. Solve for x $(0 \le x < 2\pi)$ (radians).

$4 \cdot \cos^2 x = 9\cos x + 9 \Rightarrow 4\cos^2 x - 9\cos x - 9 = 0 \Rightarrow$

$(4\cos x + 3)(\cos x - 3) = 0 \Rightarrow$

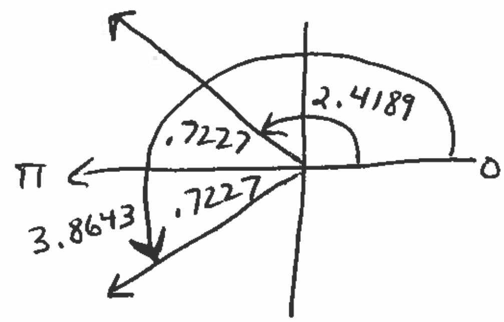
$4\cos x + 3 = 0$ or $\cos x - 3 = 0 \Rightarrow \cos x = -\frac{3}{4}$ or $\cos x = 3$.

Case 1: $\cos x = -\frac{3}{4} \Rightarrow x_{ref} = \cos^{-1}\left(\frac{3}{4}\right) \approx .7227$.

Also, $\cos x < 0$ in QII, QIII.

Thus $x = \pi - .7227 \approx 2.4189$ and

$x = \pi + .7227 \approx 3.8643$



Case 2: $\cos x = 3$ is impossible since $-1 \le \cos x \le 1$ for any x.

Therefore, $x = 2.4189, 3.8643$ are the only (approximate) solutions for $0 \le x < 2\pi$.